Crown and basal area relationships of open-grown southern pines for modeling competition and growth

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Received March 13, 1991

Accepted September 16, 1991

SMITH, W.R., FARRAR, R.M., JR., MURPHY, P.A., YEISER, J.L., MELDAHL, R.S., and KUSH, J.S. 1992. Crown and basal area relationships of open-grown southern pines for modeling competition and growth. Can. J. For. Res. **22**: 341–347.

Data were collected on open-grown loblolly pine (*Pinus taeda* L.), longleaf pine (*Pinus palustris* Mill.), and shortleaf pine (*Pinus echinata* Mill.) and analyzed to provide predictive equations of crown width and maximum potential basal area growth for crown competition and growth and yield models. The measurements were taken on 115 open-grown loblolly pine trees and 76 shortleaf pines in southeastern Arkansas. The longleaf pine data consisted of 81 open-grown trees from southern Alabama, Georgia, and Florida. A circle and an ellipse were tested as geometric models of the vertically projected crown. No significant differences between the tree shapes were found based on analyses of length and azimuth of the largest crown diameter, and the circle was chosen as an appropriate model. This indicated that only the distance between trees, not their orientation to one another, need be included in models of crown competition based on crown contact. Predictive equations of mean crown width based on diameter at breast height were fitted for each species for use in models of crown competition. A Chapman–Richards growth rate function with an intercept term was fit to periodic annual inside-bark basal area growth based on initial inside-bark basal area to provide empirical estimates of maximum basal area growth rates for growth and yield modeling of the given species. Additionally, equations to predict double bark thickness as a function of diameter at breast height were fit for each species to facilitate the use of the equations with outside-bark measurements of diameter.

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Des mesures ont été effectuées sur des pins à encens (*Pinus taeda* L.), des pins à longues feuilles (*Pinus palustris* Mill.) et des pins jaunes (*Pinus echinata* Mill.) libres de croître afin de préparer des équations pour prédire la largeur de la cime, le maximum d'accroissement potentiel en surface terrière, ainsi que des modèles de compétition des cimes et des modèles de croissance et de production. L'échantillon consiste en 115 pins à encens et en 76 pins jaunes provenant du sud-est de l'Arkansas. Les données relatives au pin à longues feuilles consistent en 81 arbres libres de croître provenant du sud de l'Alabama, de la Georgie et de la Floride. Les modèles géométriques du cercle et de l'ellipse ont été évalués pour représenter la projection verticale au sol des cimes. Aucune différence significative n'est apparue dans la forme des arbres relativement à la longueur et à la direction de la plus grande largeur de la cime. Le cercle a donc été sélectionné comme étant le modèle approprié. Ceci indique que seulement la distance entre les arbres, et non leur orientation les uns par rapport aux autres, a besoin d'être incluse dans les modèles de compétition basés sur le contact des cimes. Des équations de régression de la largeur moyenne des cimes en fonction du diamètre à hauteur de poitrine ont été ajustées pour chaque espèce en vue d'une utilisation dans des modèles de compétition des cimes. La fonction de croissance de Chapman–Richards incluant une constante a été ajustée au taux d'accroissement annuel de la surface terrière sans écorce en fonction de la surface terrière initiale sans écorce afin de fournir des estimés empiriques du taux d'accroissement maximum en surface terrière pour des fins de modélisation de la croissance de ces espèces. Par ailleurs, des équations présentant la double épaisseur de l'écorce en fonction du diamètre à hauteur de poitrine ont été calculées pour chaque espèce afin de faciliter l'utilisation des équations avec les mesures de diamètre avec écorce.

[Traduit par la rédaction]

Introduction

Loblolly pine (*Pinus taeda* L.), longleaf pine (*Pinus palustris* Mill.), and shortleaf pine (*Pinus echinata* Mill.) are commercially important species in the southern and southeastern United States. Two factors important in stand management decisions of these species are crown closure and growth rates. Open-grown trees (OGTs) can provide empirical maximum growth rates and maximum dimensions for individual stand-grown trees. These maximums have been used for the development of management guidelines and the modeling of forest growth.

Printed in Canada / Imprimé au Canada

TABLE 1. Descriptive statistics of open-grown trees by species

	Variable	Min.	Max.	Mean	SD
Shortleaf pine (*n*=75)	DBH (cm)	2.5	65.5	32.2	17.7
	Total ht. (m)	2.4	15.3	8.8	3.4
	Age (years)	3.0	50.0	21.9	11.7
	MCW (m)*	1.2	13.7	7.0	3.4
Longleaf pine (*n*=81)	DBH (cm)	2.3	65.8	23.1	17.0
	Total ht. (m)	1.8	20.4	9.8	5.3
	Age (years)	1.0	75.0	17.1	15.4
	MCW (m)*	0.6	16.4	6.0	4.4
Loblolly pine (*n*=115)	DBH (cm)	2.8	92.7	18.5	26.4
	Total ht. (m)	2.4	23.7	10.6	3.9
	Age (years)	3.0	74.0	25.6	15.5
	MCW (m)*	1.4	20.0	9.8	4.5

*MCW is the arithmetic mean of the two crown widths measured on a tree.

OGT crown widths (CWs) have been used extensively for the modeling of competition and crown closure. Krajicek *et al.* (1961) developed the crown competition factor index of relative density. Crown competition factor is based on the strong correlation of OGT CW with diameter at breast height (DBH). Crown competition factor is the sum of the projected crown area of a stand of trees predicted from the CW–DBH relationship of OGTs divided by the total stand area. The projected crown area of a tree was assumed to be circular. Crown competition factor is an index indicating the departure of the stand as a whole from an open-grown state. Strub *et al.* (1975) found crown competition factor to be a useful competition index and management tool for loblolly pine. They found stand mean diameter growth rate to be lower than the OGT diameter growth rate for equal-aged trees 1 year after a crown competition factor of 100% was reached in a stand. Smith (1987) and Nance *et al.* (1988) used the OGT CW–DBH relationship to improve the ability of weighted area potentially available (Moore *et al.* 1973) to predict basal area growth in loblolly pine. Weighted area potentially available is a polygon created around a tree based on its size relative to its neighbors' size, and position. The shape and size of the polygon indicates the tree's competitive status. The OGT CW–DBH relationship was used to identify a tree's competitors and constrain the radius of a tree's weighted area potentially available when competitors were not present in a given direction.

Basal area growth rates derived from OGTs provide reasonable maximums for constraining basal area growth rate functions derived from stand-grown trees. The maximum potential growth adjusted by a modifier accounting for competition has been used to model DBH growth in mixed species stands in the North Central States (Hahn and Leary 1979; Belcher *et al.* 1982). The maximum potential growth rates were obtained from stand-grown dominant and codominant trees of a given DBH class. Dominant and codominant trees grow less at breast height than OGTs owing to reductions in crown length and foliage from competition. OGT basal area growth rates provide an unbiased estimate of the maximums for similar modeling approaches due to breast height being within the live crown or very close to the base of the live crown (Duff and Nolan 1953; Larson 1963). The basal area growth rates of OGTs also provide useful empirical bounds that a basal area growth rate function developed for stand-grown trees should not exceed when extrapolation is necessary.

The objectives of this research were to develop predictive equations of CW and basal area growth for open-grown loblolly, longleaf, and shortleaf pine to be used in crown competition and growth and yield models of these species.

Data

Loblolly pine, longleaf pine, and shortleaf pine were selected for study because of their commercial importance in the southern United States. Measurements were taken on 115 open-grown loblolly pine trees and 76 shortleaf pines in southeastern Arkansas. The longleaf pine data set consisted of 81 OGTs from southern Alabama, Georgia, and Florida. Each tree selected had a straight bole and a uniform crown close to the ground and did not have lower branches significantly shorter than its upper branches, which would have indicated previous crown competition. To further verify that each tree was open grown throughout its life, each tree was cored and the rings were examined for growth suppression from crown competition. It was impossible to determine if seedling growth rates were influenced by herbaceous competition because the cores were taken at breast height.

The widest part of the crown (CW_{max}) and the diameter perpendicular to CW_{max} were calculated by measuring the two respective radii of each crown diameter. The azimuth of CW_{max} was recorded. Radial growth and total age were determined by a pair of cores taken at breast height, parallel and perpendicular to CW_{max}. Radial growth was recorded in 5-year intervals. A digitizer was used to record the interval length for the longleaf pine cores, and the loblolly and shortleaf pine cores were measured with a ruler. All cores were remeasured to verify the measurements. CWs, total height, height to the base of the live crown, and height to CW_{max} were measured in 0.3-m increments. DBH was measured to the nearest 0.25 cm and 5-year radial growth to the nearest 0.25 mm. When CWs were plotted against DBH, one shortleaf pine was found to have an unusually small CW for its DBH. This indicated previous crown competition, and the tree was not used in the analysis for this reason. The descriptive statistics for DBH, total height, age, and mean CW (MCW) for all species are given in Table 1.

Vertical crown projection: circle or ellipse?

The vertical projection of an OGT's crown has been assumed to be circular (Krajicek *et al.* 1961; Strub *et al.* 1975; Nance *et al.* 1988). The use of a different geometric model of a vertically projected OGT crown (i.e., an ellipse) could alter the level of crown competition as well as the crown competitors predicted from crown competition models (e.g., crown competition factor, weighted area potentially available) depending on the ellipse's orientation. To validate this assumption, the azimuth and CW data were analyzed. It was hypothesized that owing to greater light availability (southern exposure) or lower temperature stress (northern exposure) the CW of an OGT would be greater along a north–south axis. Thus, an ellipse was compared with a circle as an alternative geometric model of an OGT's crown projection.

The CW_{max} data were broken into eight azimuth classes with bearings ranging from east–west to east–northeast west–southwest. As can be seen in Fig. 1, the CW_{max} lay more frequently along a north–south direction. The frequency distribution of CW_{max} by azimuth class was tested for

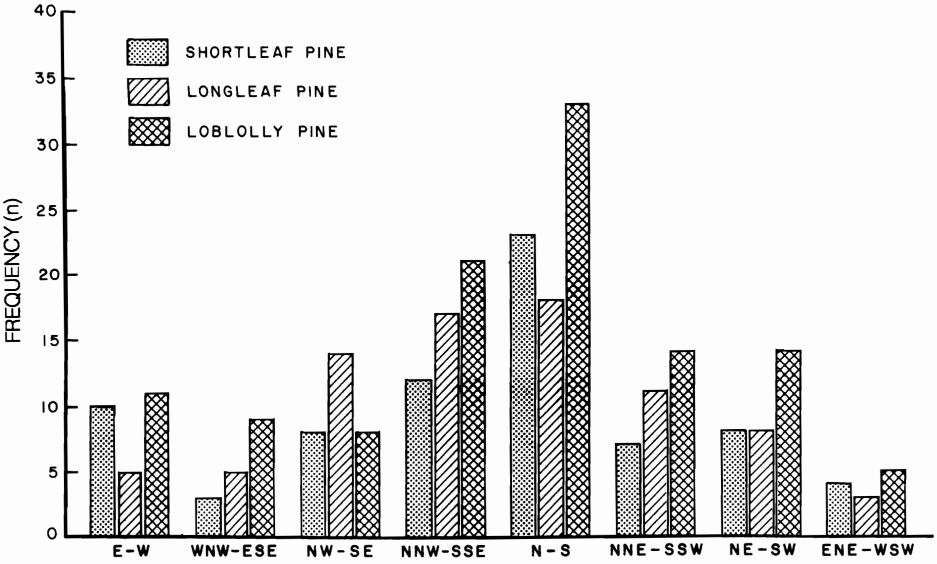

FIG. 1. The frequency distribution of the maximum crown width by bearing.

randomness with a χ^2 goodness-of-fit test to evaluate the proposed hypothesis. The χ^2 statistics for all species indicated a lack of fit to the random distribution (all $p < 0.005$) and a possible relationship between azimuth and CW_{max}.

To test if azimuth was related to CW_{max} a weighted regression analysis using dummy variables for azimuth class was conducted. DBH was used as a covariate to adjust for the relationship of tree size to CW_{max}. The plots of CW_{max} against DBH suggested a linear relationship for the longleaf pine data and a curvilinear relationship for the shortleaf and loblolly pine data. All data sets were heteroscedastic and were weighted in proportion to the square root of DBH. A variety of linear and nonlinear models were examined for fit to the data sets. One nonlinear model examined was used by Farr *et al.* (1989) to fit MCW to DBH for western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) and Sitka spruce (*Picea sitchensis* (Bong.) Carr.):

$$[1] \quad CW_{max} = \beta_0 + \beta_1 DBH^{\beta_2}$$

where β_i is a parameter estimate and $i = 0, 1$, or 2. This model gave negative intercepts when fit to the pine data sets. CW–DBH equations should have positive intercepts and for this reason were not corrected by forcing them through the origin. Instead, a linear equation was used to model the CW_{max}–DBH relationship of longleaf pine:

$$[2] \quad CW_{max} = \beta_0 + \beta_1 DBH$$

where β_i is a parameter estimate and $i = 0$ or 1. A quadratic term, $\beta_2 DBH^2$, was added to the linear equation to model the curvilinear relationship of CW_{max} to DBH for shortleaf and loblolly pine. The linear relationship between CW_{max} and DBH has also been found in loblolly pine (Strub *et al.* 1975) and radiata pine (*Pinus radiata* D. Don) (Leech 1984) data sets. The coefficients of the equations, β_0, β_1 and β_2, were estimated by ordinary least squares. The adjusted R^2 for the shortleaf equation was 0.93, and the adjusted R^2 for the longleaf and loblolly equations was 0.96. Dummy variables were included for seven azimuth classes to compare the CW_{max} in these classes with the CW_{max} with an azimuth running north–south. All the dummy variables for all species were insignificant (all $p \geq 0.086$).

To compare directly how these geometric models would influence the projected crown area of an OGT, the ratio of the area of a circle calculated from the mean radius to the area of an ellipse (Θ) was calculated. This ratio can be expressed by

$$[3] \quad \Theta = \frac{\dfrac{CW_{max}}{CW_{90°}} + \dfrac{CW_{90°}}{CW_{max}} + 2}{4}$$

where $CW_{90°}$ is the crown width 90° to CW_{max}. Θ is also the squared ratio of the arithmetic MCW to the geometric MCW. As can be seen by expressing the ratio of the areas in this manner, it would take relatively large differences in CWs to produce a significant difference in the projected crown area. Θ was plotted against DBH, height, and age for each species. Five outliers were found for the longleaf pine data set. The outliers resulted from young trees with small crown widths, 2–7 ft (0.6–2.1 m), and small absolute differences in CW, 2–3 ft (0.6–0.9 m), but large relative differences. These outliers also exerted a strong influence on the analyses because they were at the lower extreme of the range of variables and were excluded from this portion of the analysis. Correlation coefficients were calculated for the four variables by species and all were found to be insignificant (all $p \geq 0.08$). The mean of Θ and its 95% confidence intervals were calculated for each species because Θ was not a function of tree size or age. These values were 1.0048 ± 0.0014 for shortleaf pine, 1.0067 ± 0.002 for longleaf pine, and 1.0045 ± 0.0012 for loblolly pine. Although the mean Θ for each species was significantly different from one (all $p < 0.001$), the mean difference in the calculated crown projection areas using the two different geometric models was less than 1%. Based on these results a circle was judged a valid geometric model of the projected crown of an OGT and was used to calculate projected crown area.

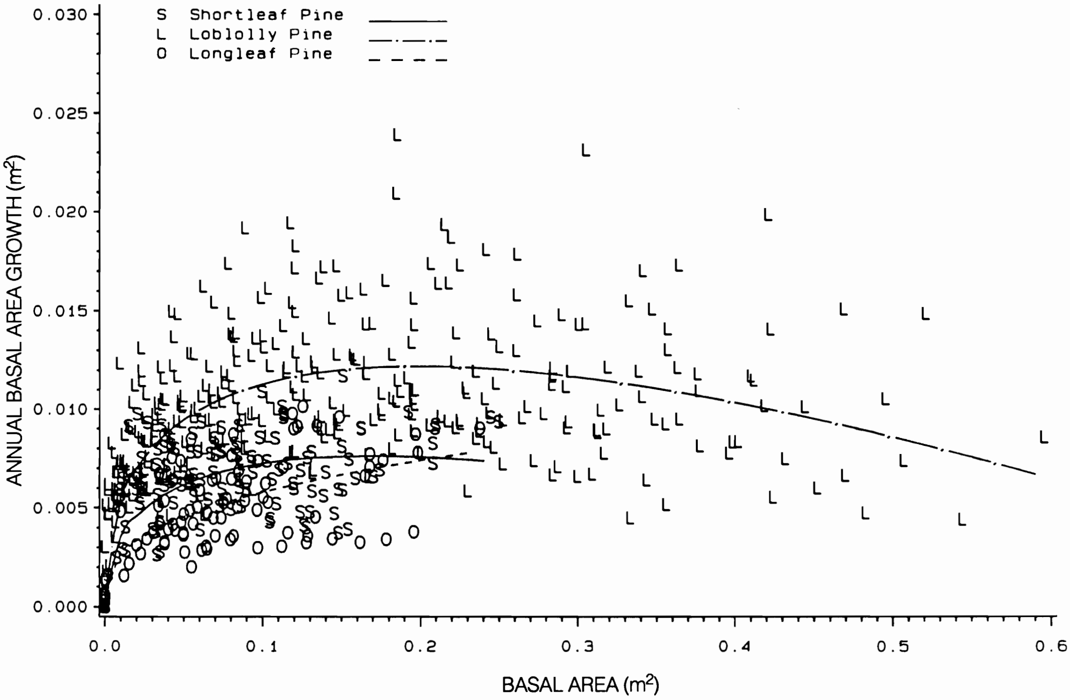

FIG. 3. The relationship of inside-bark basal area growth and inside-bark basal area for all species.

more northern locations. Sun angle is the primary controller in the orientation of shading between or within crowns. In the latitudes closer to the equator shading is primarily vertical, resulting in symmetric crown growth, while northern latitudes with lower mean sun angles have a great deal more shading and light competition caused by leaf area on the southern side of the tree (Gates 1980; List 1984), resulting in branch growth as a function of azimuth. It should then be expected that azimuth is not as important a variable for the modeling of crown and light competition for southern species as northern species in the Northern Hemisphere.

An alternative explanation for the symmetric crown growth is that there is insufficient leaf area on an OGT to produce sufficient shading to influence branch growth by orientation. The causes presented for the observed symmetric crown growth can only be hypothesized based on this data set.

The insignificant relationship of crown growth of southern pine OGTs to direction combined with the insignificant differences of OGT crown areas calculated using a circle and an ellipse resulted in the use of a circle as the geometric model. This implies for the measurement of a southern pine OGT crown that simple measurement methods, such as measuring the crown in the cardinal directions, should provide accurate estimates of CW and crown area. If the small differences of crown area using a circle versus an ellipse are of concern, the geometric MCW should be used to calculate projected crown area. The geometric MCW will be equal to the arithmetic MCW when the crown is circular.

The basal area growth relationships based on basal area for each species were fit, and the equality of relationships between species was examined and found unequal. It was interesting to notice that the longleaf pine relationships differ significantly from the shortleaf and loblolly pine relationships. Based on these equations loblolly pine has the highest growth rate, achieving a maximum periodic annual inside-bark basal area growth at 50.54 cm inside-bark DBH. Shortleaf pine achieved a maximum periodic annual inside-bark basal area growth at 46.73 cm inside-bark DBH. The maximum periodic annual inside-bark basal area growth for longleaf pine was not observed; however, it must be achieved at an inside-bark DBH greater than 54.35 cm. It is impossible to separate the influence of regional environmental factors from genetic differences on these relationships. A data set gathered from throughout the range of each species would be necessary to determine the influence of genetic and environmental factors on tree growth and form.

The equations presented should provide the tools necessary to incorporate new modeling approaches for growth and yield estimation and to assist in the determination of proper density and thinning regimes for the three southern pines studied.

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