
Basal Area Growth of Individual Trees: A Model Derived from a Regional Longleaf Pine Growth Study

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ABSTRACT. An individual tree basal area increment (BAI) model for thinned, even-aged stands of naturally regenerated longleaf pine (*Pinus palustris* Mill.) is presented. The model is an intrinsically nonlinear equation, which is constrained so that it performs within the bounds of biologically reasonable outputs for any combination of values for the independent variables. All parameters in the equation were estimated simultaneously. This is a departure from the more traditional potential-times-modifier approach in which parameters for a potential growth function are estimated from a sample of trees exhibiting the fastest growth. Independent variables used to describe BAI are stand basal area, the competitive position of an individual tree within the stand calculated as the sum of the basal areas of all trees larger than the subject tree, mean age of dominant and codominant trees, and individual tree diameter outside-bark at breast height. Noticeably absent from the model is an independent variable that explicitly characterizes site differences. FOR. SCI. 40(3): 528-542.

LONGLEAF PINE (*Pinus palustris* MILL.) STANDS cover 3.7 million acres in the southeastern United States. Some 90% of these stands are natural in origin; the remainder are planted longleaf pine (Kelly and Bechtold 1989). Natural stands of longleaf pine are an important source of high-value wood products and provide unique multiple-use benefits (Boyer 1991).

From 1964 to 1967, the USDA Forest Service established the Regional Longleaf Growth Study (RLGS) (Farrar 1989) to obtain a database for the development of growth and yield models for naturally regenerated, even-aged longleaf pine stands. Permanent sample plots were installed to cover a range of ages, densities, and site qualities. From that time, plots have been added to the study, and remeasurement has taken place regularly. Farrar (1985) used data from this study to develop stand level volume and growth predictors for longleaf pine. Somers and Farrar (1991) used the data for fitting simultaneous stand level basal area projection and survival prediction equations. This paper presents an individual tree annual basal area growth model for longleaf pine. The model forms part of a system of individual tree models for thinned, even-aged stands of naturally regenerated longleaf pine.

One approach to individual tree growth modeling is to predict growth from a potential growth function multiplied by a modifier function:

$$GROWTH = POTENTIAL\ GROWTH \times MODIFIER \quad (1)$$

The potential growth function represents the maximum growth attainable for a tree. The modifier function represents deviations from the potential due to competition. Potential growth function parameters are generally estimated separately from the modifier function. This requires that a population of trees to represent potential growth be defined. The definition of this population has varied from open growth trees (Amateis and Burkhart 1989) to some proportion of the fastest growing trees present in the data. For example, Teck and Hilt (1991) divided their data into groups defined by various combinations of stem diameter and site index. Within each group, individual trees were sorted in descending order according to their mean annual periodic basal area growth rates. The top 10% of trees in each group were used to develop the potential growth function.

Apart from the arbitrary nature of the definition of trees representing maximum attainable growth, the fact that trees cannot grow faster than a potential estimated from a small portion of the data, may be viewed as a limitation. In addition, it is difficult to assess the statistical properties of the parameters of the complete growth equation when all parameters are not estimated simultaneously.

An alternative approach is to develop a single equation to describe growth with all parameters estimated simultaneously. This approach generally relies on the formulation of intrinsically linear equations. Competition and vigor are used to explain deviations about a mean growth rate, instead of deviations from potential growth (Wykoff 1990). Recent samples are found in Hann and Larsen (1991) and Wykoff (1990).

The individual tree annual basal area increment model developed for longleaf pine resembles the potential-times-multiplier approach. However, there are some important differences. All parameters of the model were estimated simultaneously, thereby avoiding the need to define a subsample of trees for estimating the parameters of a separate potential growth equation. The model maintains ease of interpretation and is constrained so that it performs within the bounds of biologically reasonable outputs for any combination of values for the independent variables. Noticeably absent from the model is any variable to explicitly characterize site differences.

In this article, the data set used for model development and parameter estimation is described. A discussion of the model structure and properties follows.

DATA

The study used data from 190 permanent one-fifth ac and 12 one-tenth ac measurement plots from the RLGS. Plots were located in central and southern Alabama, southern Mississippi, southwest Georgia, and northern Florida. Plots were selected to fill cells in an array formed by five stand age classes with class midpoints ranging from 20 to 100 yr, five site-index classes with class midpoints ranging from 50 to 90 ft at 50 yr, and five basal area classes with midpoints ranging from 30 to 150 ft²/ac. Table 1 shows the original and final distribution of plots used in this study.

The circular measurement plots were surrounded by a 33-ft wide strip which received the same treatment as the measurement plot. An untreated, 33-ft wide, protective buffer strip surrounded the treated area. Plots were inventoried at intervals ranging from 2 to 5 yr, with 93% of the plots inventoried at 5-yr inter-

TABLE 1.

Summary of plots by age, site index (base age 50), and basal area class. Original refers to plots installed and measured from 1964 to 1966. Final refers to the condition of plots at the start of the increment period for plots measured from 1989 to 1991. Some plots were added after 1966 while others were destroyed before 1989.

Age class (yr)	Site index (ft)	Basal area (ft ² /ac)												
		Original						Final						
		30	60	90	120	150	Total	30	60	90	120	150	Total	
20	50	—	—	—	—	—	—	—	—	—	—	—	—	—
	60	8	—	—	—	—	8	—	—	—	—	—	—	
	70	9	7	1	1	—	18	1	1	2	4	—	8	
	80	5	5	3	2	—	15	2	2	1	2	—	7	
	90	—	—	—	1	—	1	—	—	—	—	—	—	
		22	12	4	4	—	42	3	3	3	6	—	15	
40	50	1	2	3	2	—	8	—	—	—	—	—	—	
	60	2	2	3	2	1	10	1	2	1	1	2	7	
	70	3	2	4	2	1	12	4	2	1	3	1	11	
	80	2	4	3	4	3	16	4	7	8	7	4	30	
	90	3	3	4	2	2	14	2	1	—	—	1	4	
		11	13	17	12	7	60	11	12	10	11	8	52	
60	50	1	1	1	1	1	5	—	2	2	1	—	5	
	60	3	3	1	1	2	10	2	—	1	3	2	8	
	70	2	4	3	6	2	17	3	3	2	2	—	10	
	80	3	3	3	3	2	14	5	6	3	4	2	20	
	90	1	1	1	1	—	4	—	—	5	1	1	7	
		10	12	9	12	7	50	10	11	13	11	5	50	
80	50	1	1	1	1	—	4	2	1	1	—	—	4	
	60	—	—	1	—	—	1	3	1	2	1	3	10	
	70	3	3	4	2	—	12	2	4	2	2	1	11	
	80	—	—	1	—	—	1	4	1	2	5	2	14	
	90	—	—	—	—	—	—	—	3	1	1	—	5	
		4	4	7	3	—	18	11	10	8	9	6	44	
100	50	—	—	—	—	—	—	—	1	1	1	—	3	
	60	—	—	—	—	—	—	1	2	2	1	—	6	
	70	—	—	—	—	—	—	4	4	2	2	3	15	
	80	—	—	—	—	—	—	—	—	—	—	—	—	
	90	—	—	—	—	—	—	—	—	—	—	—	—	
		—	—	—	—	—	—	5	7	5	4	3	24	
Totals		47	41	37	31	14	170	40	43	39	41	22	185	

vals. Measurements were made during the dormant season (October through March).

The following data were recorded for each tree in the measurement plot with diameter at breast height (dbh) greater than 0.5 in.: (1) azimuth and distance from

plot center; (2) dbh; (3) crown class defined as dominant, codominant, intermediate, or suppressed; and (4) fate of the tree defined as live or dead. In addition, a systematic subsample of trees from each 1-in. dbh class was selected at the time of plot installation. Height to the live-crown base and total height were recorded for each tree in the subsample. Age was determined for dominant and codominant subsample trees. A unique characteristic of longleaf pine is that seedlings remain virtually stemless for 2 to 7 or more years, depending on growth conditions (Boyer 1991). This stemless period is called the "grass stage." Age from seed germination is, therefore, highly dependent on the number of years spent in the grass stage. To overcome this problem, age in years was defined as the number of annual rings at 4 ft above the groundline plus 7. An increment borer was used to extract cores for annual ring counts. Cores were extracted at 4 ft to avoid interference with future dbh measurements.

At the time of installment, a target basal area was assigned to each plot. Following each measurement, plot basal area was calculated. If plot basal area was 7.5 ft²/ac or greater than the target basal area, the plot was thinned from below to reduce basal area to the target basal area. The smallest and most suppressed trees were cut, unless a larger tree had serious defects such as a severe canker, insect damage, or lightning damage. Spacing was also an important consideration in selecting trees to be cut.

Basic statistics for the dataset are given in Table 2. The data set contained 57,000 growth observations on 15,000 trees.

MODEL STRUCTURE

Model development was based on graphical analysis, theoretical assumptions about growth, goodness of fit of equations to the data, and reasonable growth projections at both an individual tree and stand level. We investigated diameter increment and basal area increment models. West (1979) concluded that no *a priori* reason existed for expressing growth as diameter increment or basal area increment. We found relationships between independent variables and basal area increment that could be expressed with known forms of biologically reasonable models. Basal area increment was, therefore, selected as the dependent variable.

TABLE 2.

Basic statistics for independent variables used in the basal area increment equation.

	Mean	Minimum	Maximum	Standard deviation
Stand basal area (ft ² /ac)	76.9	0.9	167.4	38.29
Stand age (yr)	48.9	8.9	104.8	22.99
Site index base age 50* (ft)	72.0	46.4	92.9	11.24
Tree dbh (in.)	5.8	0.4	21.5	3.59
Tree basal area larger (ft ² /ac)	53.0	0.0	166.4	36.13
Tree crown ratio	0.44	0.0	0.92	0.14
Tree basal area increment (in ² /yr)	1.16	0.0	12.19	1.07

* Site index statistics were calculated for stands over 35 yr old.

The final model described individual tree annual basal area increment (*BAI*) as a function of: (1) stand basal area (*BA*), (2) the competitive position of an individual tree within the stand expressed as the sum of the basal areas of all trees larger than the subject tree (*BAL*), (3) mean age of dominant and codominant trees (*AGE*), and (4) individual tree diameter outside-bark at breast height (*DBH*). *BAI* was calculated as:

$$\frac{\pi}{4}((DBH + DBHI)^2 - DBH^2) \quad (2)$$

where *DBHI* is the average annual diameter increment over the increment period. The assumption is that diameter increment was constant over the measurement period.

The general form of the model is:

$$BAI = BA \text{ submodel} \times BAL \text{ multiplier} \times DBH-AGE \text{ multiplier} \quad (3)$$

All parameters of the overall equation were estimated simultaneously. However, the model is best described by focusing on separate submodels before discussing the properties of the overall model. Individual submodels are illustrated based on parameters estimated for the complete equation.

The first submodel describes *BAI* as a function of stand *BA*:

$$BA \text{ submodel} = a_0 e^{a_1 BA^{0.5}} \quad (4)$$

This submodel, illustrated in Figure 1a, ensures that *BAI* of an individual tree increases as stand *BA* decreases if *BAL*, *AGE*, and *DBH* remain constant. The term a_0 is the *y*-axis intercept and a_1 the rate of decrease.

BAL was selected as the second independent variable to enter the model. *BAL* entered as a multiplier with the range of possible values restricted by the submodel form between zero and one.

$$BAL \text{ multiplier} = e^{b_0 BAL} \quad (5)$$

Equation (5) is illustrated in Figure 1b. This submodel describes the effect of *BAL* on the *BAI* of an individual tree if *BA*, *AGE*, and *DBH* remain constant. As the competitive position of a tree within the stand improves, *BAL* decreases, the *BAL* multiplier increases, and *BAI* increases.

AGE was selected as the third independent variable to enter as a second multiplier restricted by the submodel form to values between zero and one.

$$AGE \text{ multiplier} = e^{\beta_1 AGE} \quad (6)$$

This is a negative exponential equation of *AGE*. As stand *AGE* decreases, the *AGE* multiplier and *BAI* increase, if *BA*, *BAL*, and *DBH* remain constant. The parameter β_1 determines the rate at which the *AGE* multiplier approaches zero. β_1 was a function of individual tree *DBH*:

$$\beta_1 = c_1(1 - e^{c_2 DBH}) - c_0 \quad (7)$$

Equation (7) is illustrated in Figure 1c. Negative c_0 is the *y*-axis intercept, c_1 -

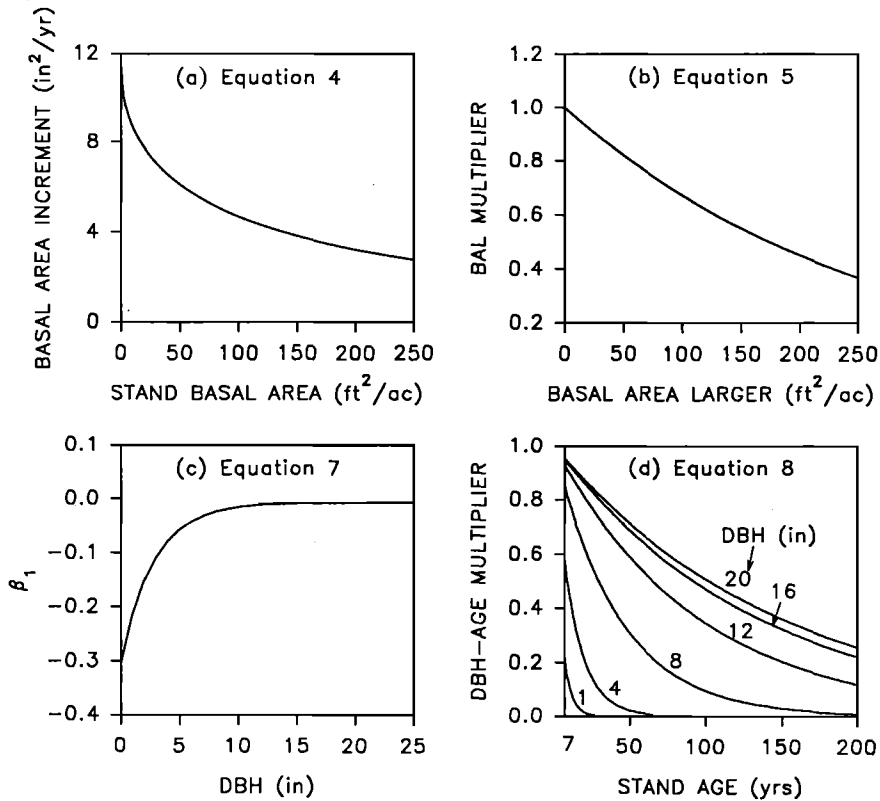


FIGURE 1. Components of the individual tree annual basal area increment equation: (a) illustrates Equation (4) which describes the effect of stand basal area, (b) illustrates Equation (5) which describes the effect of basal area larger (a measure of the trees competitive position within the stand), (c) illustrates Equation (7) which describes how the shape of the *DBH-AGE* multiplier changes with *DBH*, and (d) illustrates Equation (8) which describes the effect of historical vigor.

c_0 the upper asymptote, and c_2 the rate at which the asymptote is reached. Equations (6) and (7) were combined to form the *DBH-AGE* multiplier:

$$DBH-AGE \text{ multiplier} = e^{(c_1(1 - e^{c_2 DBH}) - c_0)AGE} \quad (8)$$

We tried many simpler equations in place of Equation (8), including functions of average past diameter growth expressed as *DBH/AGE*. These functions did not adequately describe the relationship between historical vigor and tree growth.

Age of an individual tree was defined as 7 plus the number of annual rings at 4 ft. Mean stand age can, therefore, never be less than 7. Equation (8) is illustrated in Figure 1d for a range of *AGE* values from 7 to 200 yr. This submodel describes the effect of historical tree vigor on *BAI*. For a tree in a stand of a given age, the *DBH-AGE* multiplier increases as *DBH* increases. For trees with the same *DBH*, but growing in different stands, the *DBH-AGE* multiplier increases as stand age decreases. The complete *BAI* equation is:

$$BAI = a_0 e^{a_1 BA^{0.5}} e^{b_0 BAL} e^{(c_1(1 - e^{c_2 DBH}) - c_0)AGE} \quad (9)$$

RESULTS AND DISCUSSION

Parameters of Equation (9) were estimated using nonlinear estimation techniques. Parameter estimates and related statistics are in Table 3. Interactions among the independent variables are illustrated in Figure 2. Asymptotic standard errors may be larger than they appear because data were collected from trees in plots. The observations are, therefore, not independent. The presence of autocorrelation would also result in asymptotic standard errors smaller or larger than they appear. There was little reason to suspect, however, that residuals from one observation would be correlated with residuals from an observation of tree growth 5 yr later. Gertner (1985) compared measurement intervals ranging from 1 to 10 yr and found autocorrelation not to be a problem with 5-yr intervals.

All parameters of the *BAI* model were estimated simultaneously. This is a departure from the more traditional potential-times-modifier approach in which parameters for the potential growth function are estimated from a sample of trees exhibiting the fastest growth. *BAI* of an individual tree is considered a function of stand density and two components that reflect individual tree characteristics. The *BAL* multiplier is a measure of the competitive position of a tree within the stand, while the *DBH-AGE* multiplier is a measure of historical vigor. Historical vigor is an indication of how well a tree is suited to its microsite.

Many individual tree diameter and basal area growth models include crown ratio (*CR*) as an independent variable. We explored the use of *CR* and found a correlation between *CR* and *BAI*. The final model did not include *CR* because: (1) an examination of residual structures after including *BA*, *BAL*, *DBH*, and *AGE* indicated that little could be gained by adding *CR*; (2) the inclusion of *CR* would require the measurement of another variable; and (3) *CR* measurements were available for only 30% of the observations.

The *DBH-AGE* multiplier provides the mechanism for ensuring that *BAI* eventually decreases. Once a *DBH* of approximately 16 in. is reached, the shape of the *DBH-AGE* multiplier changes very slowly (Figure 1d) and eventually becomes nearly constant with further increases in *DBH*. The shape of the *DBH-AGE* multiplier for a given *DBH* is determined by the value of β_1 [Equation (7)]. It is clear from Figure 1c that β_1 changes very slowly as *DBH* increases for larger

TABLE 3.

Parameter estimates and related statistics for the annual basal area increment equation [Equation (9)].

Parameter	Estimate	Asymptotic std. err.
a_0	11.522633	0.11258
a_1	-0.089749	0.00093
b_0	-0.003974	0.00010
c_0	0.303020	0.00176
c_1	0.296470	0.00170
c_2	-0.357679	0.00162
MSE = 0.320		
Adjusted R^2 = 0.72		

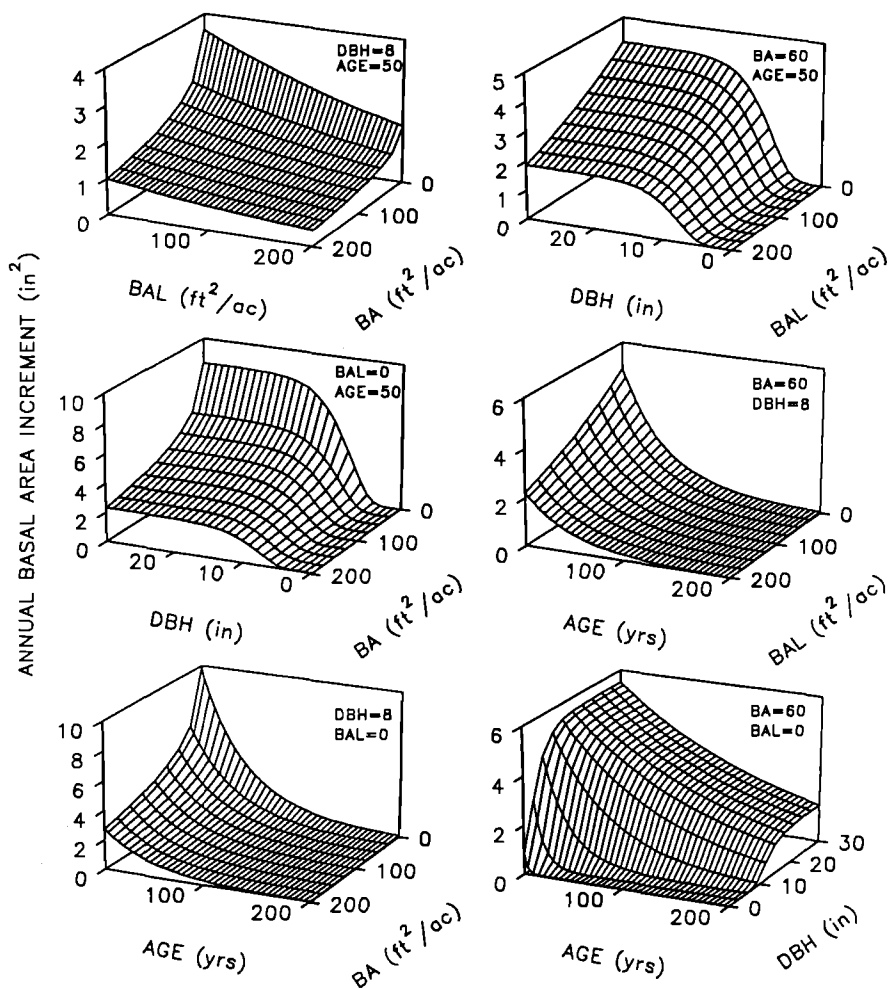


FIGURE 2. Interactions among all pairs of independent variables included in the annual basal area increment equation.

DBH classes. For large *DBH* trees, therefore, *BAI* decreases as the stand ages unless stand basal area decreases, or the competitive position of the tree within the stand improves.

To test the ability of the basal area increment model to simulate growth over time, a survival rate model was developed. Survival rate (*SR*) was modeled as a function of predicted diameter increment (*DI*) and *DBH*. Individual tree data were divided into 1-in. *DBH* classes and 0.05-in. *DI* classes. Trees in the same *DBH* and *DI* class formed a group. The aggregate survival rate of each group was calculated considering the fact that trees had been observed at different intervals (Buchman et al. 1983). An examination of survival rate trends showed that survival rate increased as *DI* increased within a *DBH* class. Furthermore, for a given *DI*, survival rate increased as *DBH* increased. These properties were incorporated into a mathematical model of survival rate. Parameters were estimated using nonlinear estimation techniques. The final equation was:

$$SR = 0.99773 - 0.20830e^{-0.52079DBH - 9.04914DI} \quad (10)$$

Equations (9) and (10) were used together to simulate growth of a hypothetical stand in order to illustrate properties of the overall model. The stand had 390 stems/ac and a basal area of 36 ft²/ac at age 20. Initial *DBH* distribution at age 20 is given in Figure 3, with illustrations of stand basal area development (Figure 3a), individual tree *DBH* development (Figure 3b), and the contribution of each diameter class to total stand basal area (Figure 3c).

Properties of the model that are clearly illustrated by Figure 3 are: (1) stand basal area reaches a maximum before decreasing (Figure 3a); (2) a tree retains its relative competitive position within a stand (Figure 3b); and (3) within a stand, larger trees grow relatively faster than smaller trees. This means that there is asymmetric competition for resources (Figure 3b and 3c).

Noticeably absent from the model is an independent variable to explicitly characterize site differences. Typically site index, mean dominant total height, or some variable derived from site index equations is included in tree growth models. An alternative approach is to directly include site characteristics. For example, Wykoff (1990) used aspect, slope, elevation, habitat type and geographic location

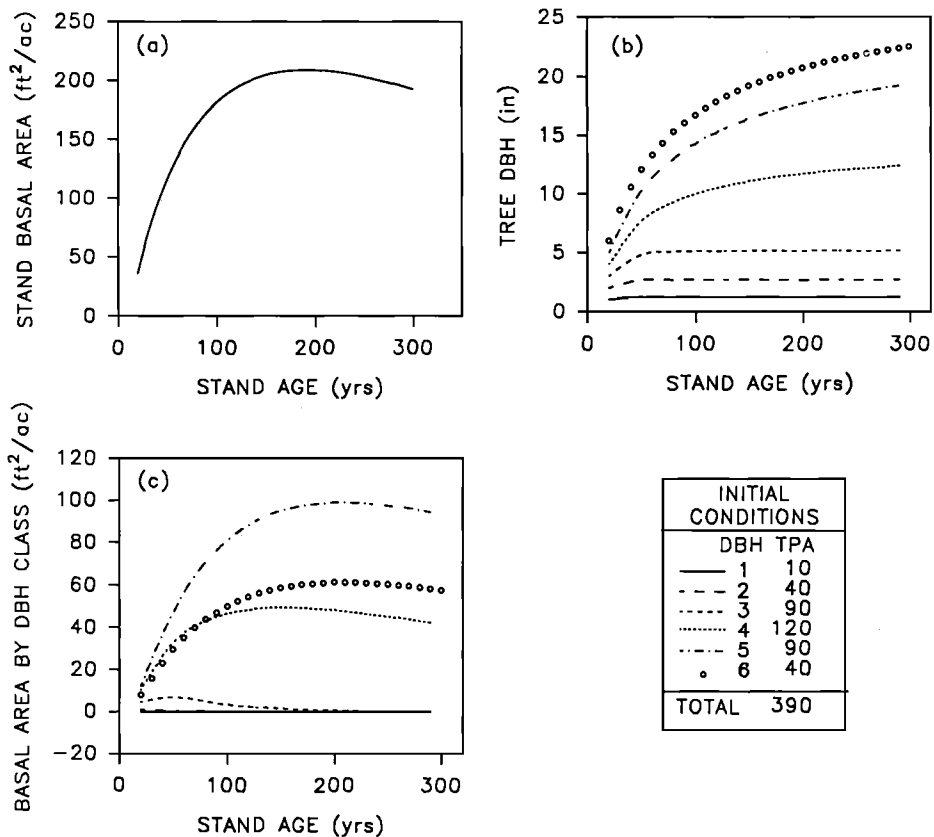


FIGURE 3. Growth simulation of a hypothetical stand using the annual basal area increment equation [Equation (9)], and the survival rate equation [Equation (10)]. The stand had 390 stems/ac and a basal area of 36 ft²/ac at age 20.

as indicators of productivity for mixed species stands in the northwestern United States. For the model presented here, a plot of residual *BAI* against site index revealed that residual *BAI* (calculated as observed value minus predicted value) did not increase as site index increased (Figure 4). Although no *a priori* decision was made to exclude site index from the model (it was a candidate independent variable) there are several benefits from doing so.

Site index is generally derived from a single observation of the mean height of dominant trees and stand age. This single observation is used to predict the mean height, at a specified index age, of the dominant trees in the stand. It assumes that a single observation of mean height and age fully describes the height growth pattern of a stand and that this height growth pattern characterizes the effect of site on the tree or stand level variable of interest. However, site index calculated from data collected at subsequent inventories of the longleaf growth plots generally showed systematic trends of steadily decreasing or increasing site indices, indicating that the height growth curves were not following the curves used to determine site index. There was, therefore, no single measure of site index to characterize site quality. McDill and Amateis (1992) characterized site index as "an indirect measure of height growth, which is, in turn, an indirect measure of site quality." Site quality is also an indirect measure of the influence of the site on the growth rate of an individual tree.

It is the inclusion of dbh and age that captures the influence of the site on individual tree growth. This result was previously reported by Lemmon and Schumacher (1962) for the diameter growth of ponderosa pine (*Pinus ponderosa* Laws.) and Cole and Stage (1972) for basal area growth of lodgepole pine (*Pinus contorta* Dougl.). In both cases the authors found that a site index term was not required once dbh and age were added to the model. Cole and Stage (1972) selected site index instead of dbh and age due to the expense incurred in boring

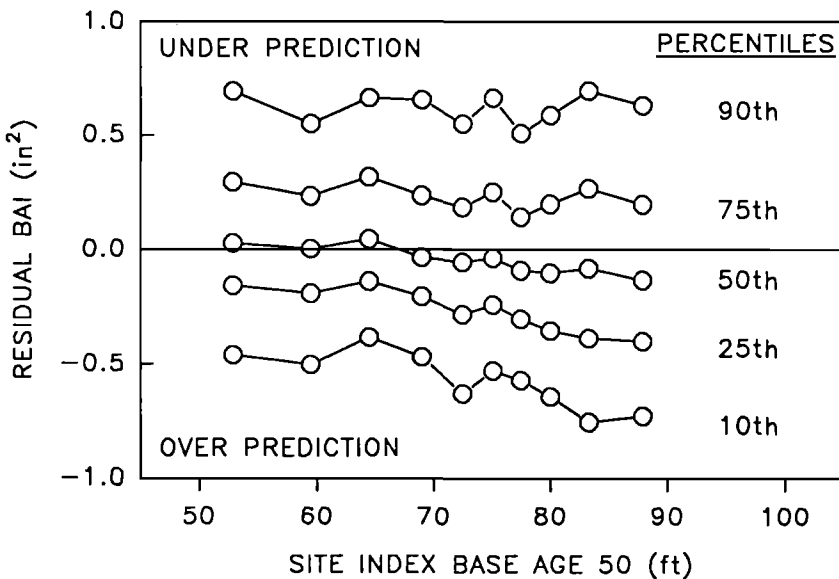


FIGURE 4. Residual annual basal area increment (calculated as observed value minus predicted value) plotted against site index base age 50.

trees. In our case, however, we use stand age, which is defined as the mean age of dominant and codominant trees. This stand age would still be required for a model that included site index.

Another feature of the model is that there is no predefined potential growth function imposed on the model. The need to define a separate population for modeling potential growth is avoided by having a single equation with all parameters estimated simultaneously. Maximum growth rates allowed by the model result from the interaction between stand *BA*, stand *AGE*, and individual tree *DBH* (the *BAL* multiplier is equal to 1 for the largest tree in a stand). The maximum value for the *DBH-AGE* multiplier for a tree of a given *DBH* is at *AGE* 7 (Figure 1d). For example, the maximum possible *DBH-AGE* multiplier for a tree with a *DBH* of 4 in. is 0.58, and the maximum possible growth rate (potential growth) is Equation (4) multiplied by 0.58. We do not expect, however, to have large trees at *AGE* 7. For example, the youngest plot in the study data set with an 8-in. tree was 20 yr old.

To illustrate potential growth of trees with different *DBH*s, *BAL* was set to zero, and *AGE* to the age of the youngest stand in the data set containing a tree with the specified *DBH*. Potential growth would be greater for younger values of

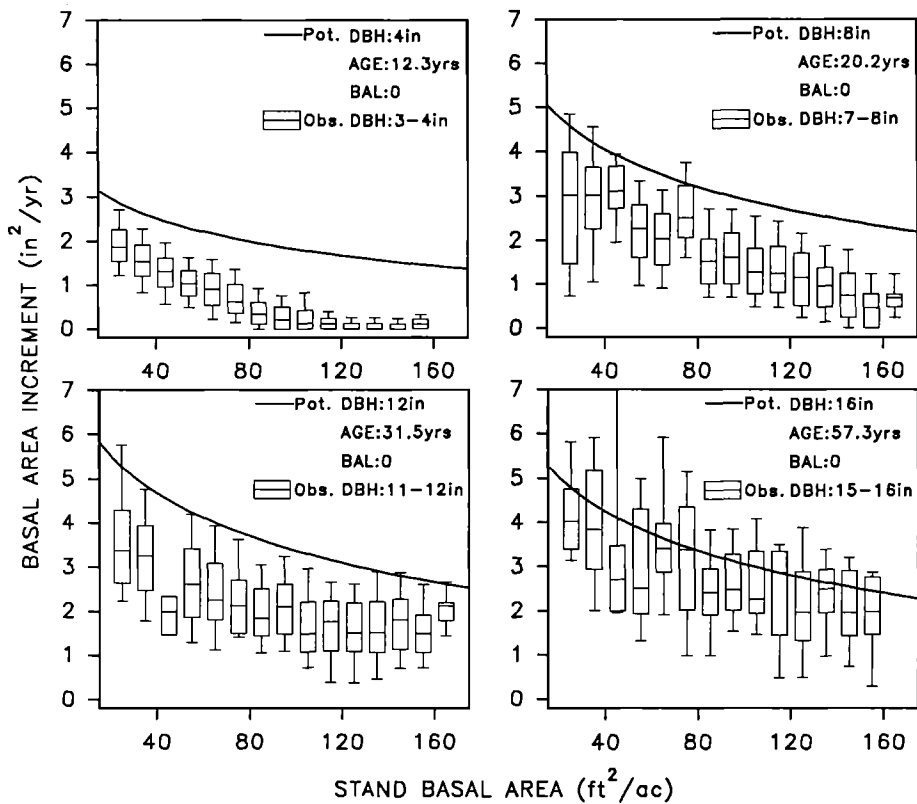


FIGURE 5. Graphs of potential growth for trees with *DBH*s of 4, 8, 12, and 16 in. overlaid on observed growth for *DBH* classes of 3–4, 7–8, 11–12, and 15–16 inches respectively. Potential growth was determined from the fastest growing tree in the data set for each *DBH* class illustrated. Observed growth is plotted as box plots for different stand basal area classes. Horizontal lines of the box plots mark the 10th, 25th, 50th, 75th, and 90th percentile points of the data.

AGE. For example, a tree that reached a diameter of 8 in. before age 20 would have a higher *DBH-AGE* multiplier and, therefore, greater potential growth.

The potential growth curves were plotted with observed growth of trees with a range of *DBHs* less than, or equal to, the specified *DBH* (Figure 5). There was a widening gap between potential growth and observed growth as stand *BA* increased for 4 in. and 8 in. trees. This was expected because few trees of this size have a small *BAL* when they occur in stands with a large *BA*. It is clear, however, that growth rates of the fastest growing 16 in. trees were underpredicted. An examination of mean plot residuals revealed that the largest underprediction of growth occurred on a group of plots located near Bainbridge, GA. With a site index of 90 ft (base age 50), these plots were on the best longleaf pine sites included in the study. However, growth rates for all trees on sites with high site indices were not underpredicted (Figure 4). A closer examination of residuals revealed that the largest underpredictions were associated with dominant, large diameter trees, on the best sites.

The ability of the model to simulate reasonable growth for open grown trees was tested. An open grown condition was imposed by having stand *BA* equal to tree basal area, and *BAL* equal to zero. The simulated *DBH* development of an open grown tree, with a *DBH* of 2 in. at age 10, is shown in Figure 6. The plot is overlaid on the observed *DBHs* of 81 open grown trees. The data on open grown trees were collected for a separate study described by Kush et al. (1988). Simulated *DBH* development provided a good approximation of the average size of the open grown trees in the data set.

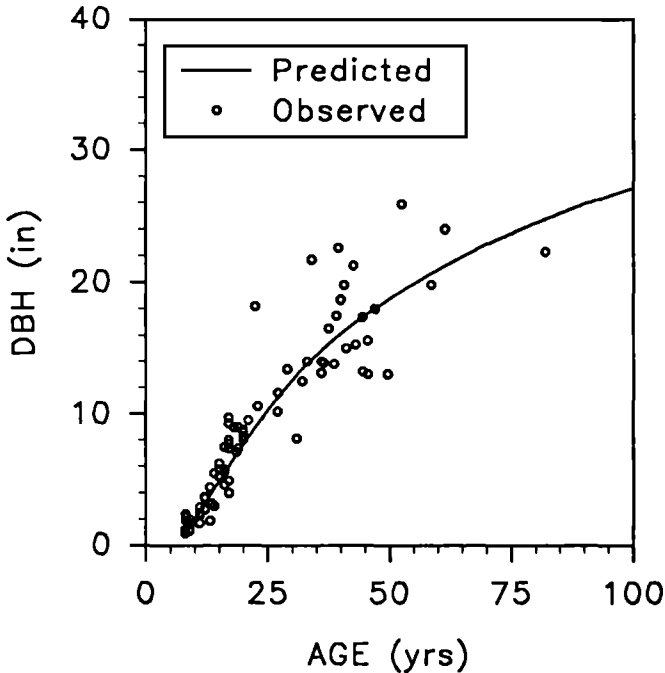


FIGURE 6. Predicted *DBH* and basal area development of an open grown tree with a *DBH* of 2 in. at age 10. Predicted *DBH* development is overlaid on a plot of observed *DBH* for 81 open grown trees.

The model was also tested by projecting growth for 10 yr using stands unthinned over a 10 yr period. Predicted *DBH* and stand *BA* were compared to observed values after projection periods of 5 and 10 yr. Table 4 lists diameter residuals for 1 in. *DBH* classes with midpoints ranging from 0.5 to 19.5 in. There was a small, but consistent, underprediction of *DBH* that averaged 0.04 in. after 5 yr and 0.08 in. after 10 yr. Underprediction increased for trees over 15 in. Average growth was underpredicted because some trees grow faster than the maximum growth rate allowed by the model (Figure 5). The average absolute residual was 0.21 in. and 0.38 in. after projection periods of 5 and 10 yr respectively. Residual statistics for stand *BA* are given in Table 5. The average absolute residual was 3.25 ft²/ac and 5.75 ft²/ac after projection periods of 5 and 10 yr, respectively.

In the model presented, growth acceleration in a thinned stand results from the reduction in stand *BA* and, for some trees, a reduction in individual tree *BAL*. The apparent paradox that the model will simulate the same future development for any two stands with identical structures after thinning, irrespective of site quality, may not be serious. It is reasonable to expect that the structure of stands with the same age, but growing on different quality sites, will be different. To make the

TABLE 4.

Diameter residuals after projection periods of 5 and 10 yr. Stands included in the projections were not thinned during the 10-yr projection interval. Residuals were calculated as observed minus predicted values.

Dbh class midpt. (in.)	5-year projection				10-year projection			
	No. obs.	Ave. residual	Ave. abs. residual	SD of residual	No. obs.	Ave. residual	Ave. abs. residual	SD of residual
			(in.)				(in.)	
0.5	8	0.06	0.12	0.12	2	0.16	0.16	0.07
1.5	103	0.05	0.12	0.16	75	0.03	0.15	0.19
2.5	164	0.08	0.21	0.32	133	0.02	0.27	0.41
3.5	206	0.04	0.19	0.29	161	0.04	0.29	0.47
4.5	254	0.04	0.20	0.26	197	0.01	0.34	0.49
5.5	262	0.08	0.21	0.26	213	0.07	0.38	0.50
6.5	298	0.08	0.24	0.30	242	0.09	0.39	0.48
7.5	327	0.08	0.23	0.31	285	0.13	0.41	0.51
8.5	324	0.04	0.24	0.30	312	0.13	0.42	0.53
9.5	343	0.02	0.22	0.28	301	0.12	0.41	0.52
10.5	301	-0.03	0.21	0.27	336	0.07	0.42	0.52
11.5	270	-0.01	0.21	0.28	255	-0.04	0.35	0.45
12.5	192	-0.01	0.22	0.27	236	0.05	0.38	0.51
13.5	175	-0.04	0.18	0.25	193	0.07	0.40	0.52
14.5	93	0.09	0.21	0.26	134	0.03	0.32	0.43
15.5	67	0.07	0.17	0.22	88	0.25	0.39	0.45
16.5	32	0.16	0.23	0.25	47	0.27	0.36	0.39
17.5	16	0.14	0.19	0.21	31	0.34	0.40	0.43
18.5	13	0.26	0.28	0.20	13	0.46	0.50	0.33
<u>19.5</u>	<u>2</u>	<u>0.26</u>	<u>0.26</u>	<u>0.23</u>	<u>8</u>	<u>0.41</u>	<u>0.43</u>	<u>0.32</u>
Overall	3450	0.04	0.21	0.28	3262	0.08	0.38	0.49

TABLE 5.

Stand basal area residuals after projection periods of 5 and 10 yr. Stands included in the projections were not thinned during the 10-yr projection interval. Residuals were calculated as observed minus predicted values.

No. obs.	Projection interval (yr)	Ave. residual	Ave. absolute residual	Std. dev. of residual
			----- (ft^2/ac) -----	
102	5	-0.20	3.25	5.96
102	10	0.02	5.75	9.20

structures identical by thinning would require a type of thinning never applied in practice.

Most sample plots used in this study were thinned one or more times over the 25-yr measurement period to maintain target basal areas. Plots of residuals against percent basal area removed or actual basal area removed, revealed no consistent patterns of under- or overprediction. Users of the model should remember, however, that we calculated *BAI* based on the assumption that diameter increment was constant over the measurement interval. There may be a lag before trees respond to the increased growing space. In this case growth would be overpredicted in the years immediately following thinning.

The individual tree basal area growth model developed for longleaf pine was an intrinsically nonlinear model that: (1) provided a good fit to the data; (2) performed within the bounds of biologically reasonable outputs for any combination of values for the independent variables; (3) maintained ease of interpretation; and (4) remained a single equation thereby avoiding the need to define a subpopulation of trees for modeling potential growth. In future years additional data will become available for large diameter trees, including diameters outside the range used for development of the current model. The ability of the model to predict growth of these large diameter trees will be evaluated and refinements made if necessary. Additional future work will concentrate on model validation using independent data sets.

LITERATURE CITED

- AMATEIS, R.L., and H.E. BURKHART. 1989. Diameter increment and survival equations for loblolly pine trees growing in thinned and unthinned plantations on cutover, site-prepared lands. *South. J. Appl. For.* 13:170-174.
- BOYER, W.D. 1991. Longleaf Pine. P. 405-412 *in* *Silvics of North America I. Conifers*. USDA Agric. Handb. 654.
- BUCHMAN, R.G., S.P. PEDERSON, and N.R. WALTERS. 1983. A tree survival model with application to species of the Great Lakes region. *Can. J. For. Res.* 13:601-608.
- COLE, M.C., and A.R. STAGE. 1972. Estimating future diameters of lodgepole pine trees. USDA For. Serv. Res. Pap. INT-131. 20 p.
- FARRAR, R.M., JR. 1985. Volume and growth predictions for thinned even-aged natural longleaf pine stands in the East Gulf area. USDA For. Serv. Res. Pap. SO-220. 172 p.
- FARRAR, R.M., JR. 1989. Predictions of volume and volume growth in naturally-regenerated longleaf

- pine stands. P. 170–192 *in* Farrar, R.M. (ed.). Proc. Symp. Management of Longleaf Pine. USDA For. Serv. Gen. Tech. Rep. SO-75.
- GERTNER, G.Z. 1985. Efficient nonlinear growth model estimation: Its relationship to measurement interval. *For. Sci.* 31(4):821–826.
- HANN, D.W., and D.R. LARSEN. 1991. Diameter growth equations for fourteen tree species in southwest Oregon. *For. Res. Lab. Res. Bull.* 69. Oregon State Univ., Corvallis. 18 p.
- KELLY, J.F., and W.A. BECHTOLD. 1989. The longleaf pine resource. P. 11–22 *in* Farrar, R.M. (ed.). Proc. Symp. Management on Longleaf Pine. USDA For. Serv. Gen. Tech. Rep. SO-75.
- KUSH, J.S., R.K. BOLTON, T.R. BOTTENFIELD, R.S. MELDAHL, and R.M. FARRAR, JR. 1988. Longleaf pine crown relationships: A preliminary analysis. P. 433–436 *in* Miller, J.H. (ed.). Proc. Fifth Bienn. South. Silv. Res. Conf. USDA For. Serv. Gen. Tech. Rep. SO-74.
- LEMMON, P.E., and F.X. SCHUMACHER. 1962. Volume and diameter growth of ponderosa pine trees as influenced by site index, density, age, and size. *For. Sci.* 8(3):236–249.
- MCDILL, M.E., and R.L. AMATEIS. 1992. Measuring forest site quality using the parameters of a dimensionally compatible height growth function. *For. Sci.* 38(2):409–429.
- SOMERS, G.L., and R.M. FARRAR, JR. 1991. Biomathematical growth equations for natural longleaf pine stands. *For. Sci.* 37(1):227–224.
- TECK, R.M., and D.E. HILT. 1991. Individual-tree diameter growth model for the Northeastern United States. USDA For. Serv. Res. Pap. NE-649. 10 p.
- WEST, P.W. 1979. Use of diameter increment and basal area increment in tree growth studies. *Can. J. For. Res.* 10:71–77.
- WYKOFF, W.R. 1990. A basal area increment model for individual conifers in the northern Rocky Mountains. *For. Sci.* 36(4):1077–1104.

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